79. Proposed by GEORGE LILLEY, Ph.D., LL. D., Professor of Mathematics, University of Oregon, Eugene, Oregon.

Find the area included between $y=\sin^{\pi} x+\cos^{e}x$; $y=\pi e(\sin^{\pi} x\cos^{e}x)$ and the length of its boundary, true to six decimal places, when $\pi=3.14159$, $e=2.7182$.

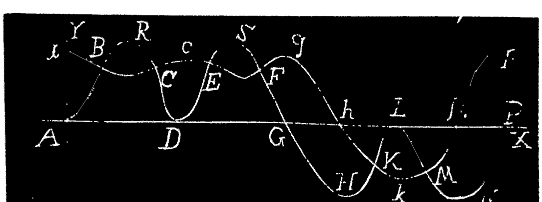
Solution by G. B. M. ZERR, A. M., Ph.D., Professor of Mathematics and Stience, Chester High School, Chester, Pa.

When $x=0°$, $\sin^{\pi} x=0$, $\cos^{e}x=1.0000$.
When $x=30°$, $\sin^{\pi} x=.1133$, $\cos^{e}x=.6764$.
When $x=45°$, $\sin^{\pi} x=.3366$, $\cos^{e}x=.3899$.
When $x=60°$, $\sin^{\pi} x=.8006$, $\cos^{e}x=.1520$.
When $x=90°$, $\sin^{\pi} x=1.0000$, $\cos^{e}x=0$.

Since e is even, $\cos^{e}x$ is always positive ; hence the following coördinates :

	$y=\sin^{\pi} x+\cos^{e}x$	$y=\pi e(\sin^{\pi} x\cos^{e}x)$
x	y	y
0°= .0	1.0000	0
30°= .5236	.7897	.6541
45°= .7853	.7265	1.1207
60°=1.0471	.9526	1.0392
90°=1.5707	1.0000	0
120°=2.0942	.9526	1 0392
135°=2.3560	.7265	1.1207
150°=2.6178	.7897	.6541
180°=3.1414	1.0000	0
210°=3.6650	.5631	−.6541
225°=3.9268	.0533	−1.1207
240°=4.1886	−.6480	−1.0392
270°=4.7121	−1.0000	0
300°=5.2360	−.6480	−1.0392
315°=5.4978	.0533	−1.1207
330°=5.7596	.5631	−.6541
360°=6.2828	1.0000	0

The curves drawn from these coördinates are somewhat like the figure, $ABRCDESFGHKLMNP$ corresponding to $y=\pi e(\sin^{\pi} x\cos^{e}x)$; $aBCEFghKkMlp$ corresponding to $y=\sin^{\pi} x+\cos^{e}x$.



The curves are indefinite in length. The areas included between the curves from $x=0°$ to $x=360°$ are $AaB + BRC + CDEc + ESF + FGHKhg + LMkK + PNMlp$. The length of the boundary is the whole length of both curves. The whole area common to both curves is infinite. The area above is but the area for one revolution. Area BRC=area ESF,

area AaB + area $PNMlp$ = area $FGHKhg$. The intersections are $x=32^\circ\ 48'$, $x=61^\circ$, $x=119^\circ$, $x=147^\circ\ 12'$, $x=244^\circ\ 22'$, $x=295^\circ\ 38'$. The integrations are exceedingly tedious, but can be performed. If 3.1416 had been used for π the curves for one revolution would have consisted of four parts each equal to $aBCc$ and $ABRCD$.

MECHANICS.

Criticism on Professor Zerr's Solution of Problem 67, Mechanics, by J. M. ARNOLD, Crompton, R. I.

I wish to take exception to Professor Zerr's solution of No. 63 Mechanics, in the May number. The preliminary reasoning and the diagram are correct, but when he proceeds to find the required angles he commences with the assumption "The $\angle ABC = \angle CDE$ and the $\angle BAC = \angle CED$." This is wrong as it can be easily shown that these angles are not equal. Therefore his result must be in error. I have not had time to solve the problem correctly, but I think it leads to very complicated equations.

70. Proposed by CHARLES E. MEYERS, Canton, Ohio.

A homogeneous sphere, radius r, having an angular velocity ω, gradually contracts by cooling. What will be the angular velocity at the instant the radius becomes $\frac{1}{2}r$?

Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy. Ohio University, Athens, Ohio.

Let m=the constant mass ; r, $\frac{1}{2}r$ the original and final radii ; ω', the required angular velocity ; k, k' the radii of gyration corresponding.

The moment of angular momentum remaining constant,

$$mk^2\omega = mk'^2\omega' \ldots\ldots (1).$$

But $k^2=\frac{2}{5}r^2$, $k'^2=\frac{2}{5}(\frac{1}{2}r)^2=\frac{1}{10}r^2$, (1) plainly gives $\omega'=4\omega$.

Also solved in the same manner by *G. B. M. ZERR*.

71. Proposed by the late B. F. BURLESON, Oneida Castle, N. Y.

Three men own a sphere of gold the density of which varies as the square of the distance from the center. If two segments be cut off each one inch from the center of the sphere it will be divided into three parts of equal value. Determine the diameter of the sphere.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let ρ=density=r^2 in this case, a=radius. Then the mass of each segment cut off is

$$M=\int_1^a\int_0^{2\pi}\int_0^{\cos^{-1}(1/a)}\rho r^2\sin\theta d\theta d\varphi dr=\int_1^a\int_0^{2\pi}\int_0^{\cos^{-1}(1/a)}r^4\sin\theta dr d\varphi d\theta$$

$$=\frac{2\pi}{5a}(a^5-1)(a-1).$$